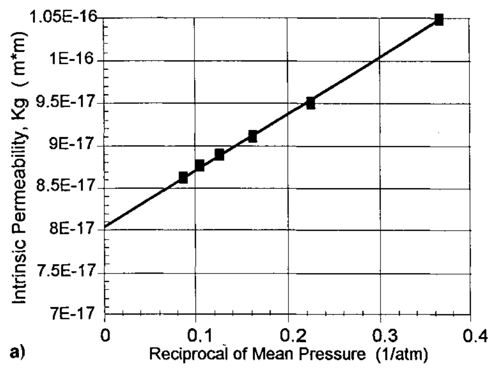
Table 3 Properties of superheated steam and intrinsic permeability of concrete mortar

Specimen temperature, °C	ρ_0, g/cm^3	$\bar{\mu}$, cP	K_g, m^2
181.7	4.82×10^{-4}	1.54×10^{-2}	9.44×10^{-17}
207.2	4.56×10^{-4}	1.61×10^{-2}	8.786×10^{-17}
215.6	4.51×10^{-4}	1.65×10^{-2}	8.687×10^{-17}
225.1	4.43×10^{-4}	1.71×10^{-2}	8.608×10^{-17}



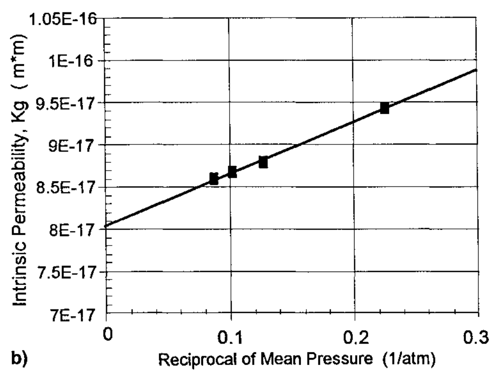


Fig. 3 Intrinsic gas permeability K_g vs reciprocal of mean pressure: a) nitrogen tests and b) superheated steam tests.

permeability K_g (Table 3) from Eq. (3), and plotted (Fig. 3b) vs the reciprocal of the mean pressure $[(P_h + P_l)/2]$. The samples' length L was 50.8 mm, and their cross-sectional area A was 126 cm^2. From the plotted data, the value of K_l (intercept) was determined to be 8.04×10^{-17}, and Klinkenberg constant b (slope/K_l) was found to be 0.77 with an $R^2 = 0.99$. Note the value of K_g varied by 20% over the pressure ranges tested, suggesting that a constant K as in Darcy's expression [Eq. (1)] is inappropriate. Also, note that the difference in K_l for both nitrogen and superheated steam data differed by less than 1%, and the b value for both gases differed by only 4%.

Presented in Fig. 4 is the pressure distribution measured across a sample in a nitrogen test with a feed pressure of 10 atm. Evident from the figure, the pressure drop across the specimen is not linear. An analytical expression may be derived by substituting the volume flow rate equation [Eq. (2)] into the conservation of mass expression and solving for pressure. For one-dimensional steady-state conditions, the resulting pressure P is

$$(P + b)^2 = [(P_l + b)^2 - (P_h + b)^2](x/L) + (P_h + b)^2 \quad (4)$$

where x is the distance from the high-pressure boundary, b is Klinkenberg's constant, and P_h and P_l are the high and low pressure at the boundaries, respectively. Plotted as a solid

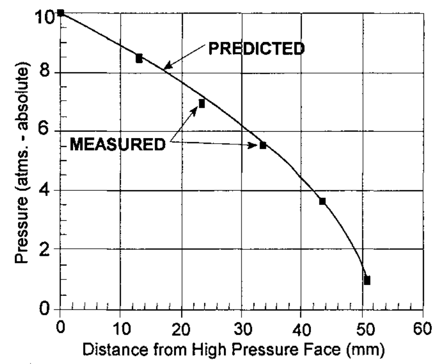


Fig. 4 Pressure profile across the specimen.

line in Fig. 4 is the solution of Eq. (4) with a Klinkenberg's coefficient b of 0.81. There is less than a 4% error between the measured and predicted pressures across the whole sample. For a b value of 0.0 (Darcy's expression), Eq. (4) was found to be in error by 15% when compared to the measured pressure.

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Natural Convection in Horizontal-Layered Porous Annuli

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Introduction

OVER the past decades, heat transfer in saturated porous media has received considerable attention for its important applications in geophysics and energy-related engineering problems. However, previous efforts have been centered primarily on a homogeneous system, whereas a layered system, although encountered more frequently in engineering practice, has received very little attention. Most available results on heat transfer in layered porous media are limited to simple geometries like horizontal layers[1–4] or vertical layers.[5,6] In addition, their emphasis has been placed on the establishment of the criterion for the onset of convection. For a layered annulus, the results are very few and they are reported only by Muralidhar et al.[7] However, their results show

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little dependence on the permeability contrast that characterizes the system.

It is the purpose of this study to re-examine the problem and to investigate the appropriateness of using an effective permeability in the calculation of heat transfer from nonhomogeneous systems. It is expected that the results thus obtained will not only provide useful information for engineers, but also serve a complement to the existing literature.

Analysis

The geometry considered is a horizontal porous annulus comprising two sublayers. For each sublayer, it is assumed to be fully saturated and has a different permeability and thermal conductivity. For applications, the case considered here may correspond to a circular pipe that is wrapped with layers of insulation or a nuclear waste canister that is protected by both engineered and natural barriers. Typically, the inner cylinder is kept at a constant temperature T_1 while the outer cylinder is maintained at the ambient temperature T_2 ($T_1 > T_2$). The governing equations based on Darcy's law are given by

$$\frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{1}{r}\frac{\partial v_i}{\partial \theta} = 0 \tag{1}$$

$$u_i = -\frac{K_i}{\mu}\left(\frac{\partial p_i}{\partial r} + \rho g \sin\theta\right) \tag{2}$$

$$v_i = -\frac{K_i}{\mu}\left(\frac{1}{r}\frac{\partial p_i}{\partial \theta} + \rho g \cos\theta\right) \tag{3}$$

$$u_i\frac{\partial T_i}{\partial r} + \frac{v_i}{r}\frac{\partial T_i}{\partial \theta} = \alpha_i\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T_i}{\partial \theta^2}\right] \tag{4}$$

where the subscript $i = 1, 2$, denoting the inner and outer sublayer, respectively. At the layer interface, the appropriate conditions are

$$p_1 = p_2 \tag{5a}$$

$$T_1 = T_2 \tag{5b}$$

$$u_1 = u_2 \tag{5c}$$

$$k_1\frac{\partial T_1}{\partial r} = k_2\frac{\partial T_2}{\partial r} \tag{5d}$$

which are nothing but the continuity of pressure, temperature, radial flow, and heat flux. The justification of these boundary conditions is given by Rana et al.[2] as well as McKibbin and O'Sullivan.[3,4]

To solve the simultaneous equations defined previously, a coordinate transformation that maps the computational domain onto a rectangular region has been used to facilitate calculations. The numerical details are omitted here for brevity and can be found in Refs. 8 and 9. Uniform grid, 51 × 121, in the transformed domain, has been chosen for the present study. It should be noted that further grid refinement does not produce any significant improvement in the calculated Nusselt numbers. The validation of the numerical code has been reported in Ref. 9. In view of the parameters and the complexity involved in this problem, the present study is limited to the case of $R_1 = r_1/D = 1/2$ and $R_2 = r_2/D = 3/2$. In addition, the layer interface has been fixed at $r = (r_1 + r_2)/2$.

Results and Discussion

It is well-known that natural convection in a homogeneous porous annulus is established in the form of two primary cells at a moderate Rayleigh number.[7,9] Heated fluid rises to the top along the inner cylinder and, when cooled, returns to the bottom along the outer cylinder. For a layered annulus, the flow and temperature profiles are very different from those of a homogeneous one, due to the step change in permeabilities (Fig. 1). Taking advantage of the symmetrical property of the problem, isotherms are plotted on the left half of the annulus and streamlines are on the right half. For simplicity, one can envision that a step change in the permeability is created by replacing the outer portion of an originally homogeneous annulus with a different material (i.e., K_1 is held constant). When this is done by introducing a less permeable material to the outer region (i.e., $K_1/K_2 > 1$), the added flow resistance in the outer layer inhibits the penetration of the convective flow such that the recirculating cell is primarily confined to the inner layer. The outer sublayer in this case acts like a convection suppressor.

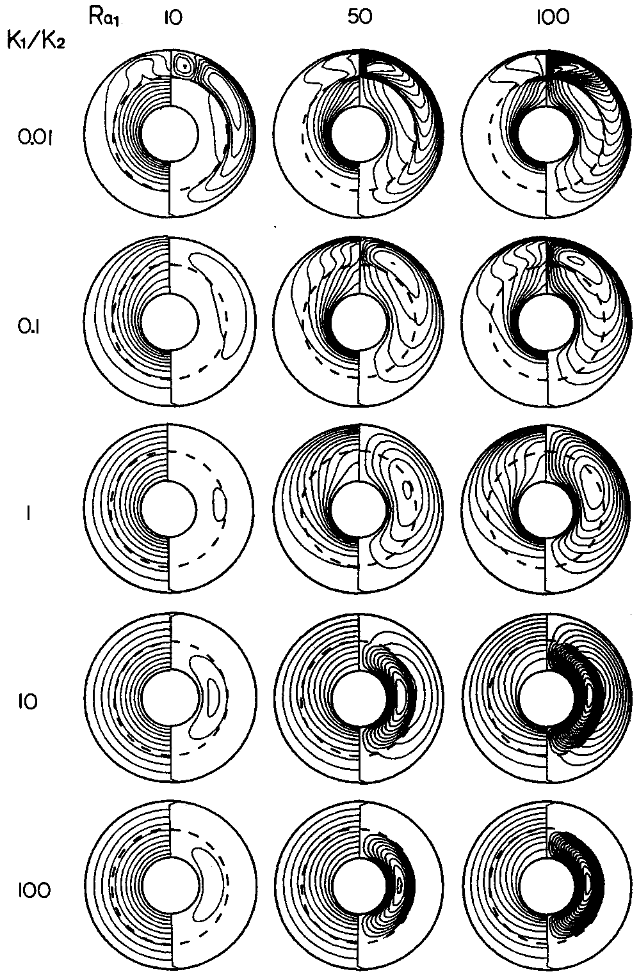

Fig. 1 Effects of permeability contrast on the flow and temperature fields of a layered porous annulus ($k_1/k_2 = 1$, $\Delta\Theta = 0.1$, $\Delta\psi = 0.5$ for $K_1/K_2 = 0.01$, 0.1, and 1, $\Delta\psi = 2$ for $K_1/K_2 = 10$ and 100).

If a more permeable material is emplaced instead (i.e., $K_1/K_2 < 1$), the convective cell becomes notably stronger when compared with that of a uniform case. Since less flow resistance is encountered in the outer layer, the buoyancy-induced flow is initiated at a smaller Rayleigh number. The outer sublayer thus acts like a convection promoter. Due to the difference in the role played by the outer sublayer, the heat transfer modes in these two cases are also distinctive. For a layered annulus with $K_1/K_2 > 1$, heat transfer is mainly by conduction at a small Rayleigh number, which is evident from the corresponding isotherms shown in Fig. 1. As the Rayleigh number increases, it changes gradually from conduction to weak convection. On the other hand, heat transfer is always by convection for a layered annulus with $K_1/K_2 < 1$.

When a system is inhomogeneous in permeability, its thermophysical properties are usually nonuniform. This is especially true when the two sublayers are composed of materials with distinct thermal properties. The nonuniformity in thermal conductivity can further complicate the flow and temperature fields as evident from Fig. 2. When the inner layer is less conductive (i.e., $k_1/k_2 < 1$), the thermal resistance across the inner layer is greater than that of the outer layer. Thus, a larger temperature gradient is experienced in the inner region. As a result, the buoyancy-induced flow becomes stronger in the inner region. As the conductivity ratio increases ($k_1/k_2 > 1$), the thermal resistance across the inner layer decreases while it is increased for the outer layer. Consequently, the strength of the convective flow in the inner region is reduced and it is increased for the flow in the outer region. With the combined effects of nonuniform permeability and thermal conductivity, it is observed that the temperature gradient (and therefore, the strength of the convective flow) in the inner region is the largest when $k_1/k_2 = 0.5$ and $K_1/K_2 = 0.1$, whereas it is the smallest when $k_1/k_2 = 2$ and $K_1/K_2 = 10$ (referring to Fig. 2).

In the design of thermal systems, the heat transfer coefficient is one of the most important pieces of information required. For the present study, the heat transfer coefficient in terms of the Nusselt number is given by

$$Nu = \frac{hD}{k_1} = -\frac{1}{R_1 \ln(R_2/R_1)} \int_{-1/2}^{1/2} \frac{\partial \Theta_1}{\partial x}\bigg|_{x=-1/2} \mathrm{d}y \tag{6}$$

However, heat transfer results for the problem under consideration are most informative if the Nusselt number thus obtained is normalized by its conduction value. In this way, the normalized Nusselt number also represents the relative importance of convection to conduction. It is easy to show that the conduction Nusselt number can be evaluated as

$$Nu_{\mathrm{cond}} = \frac{1}{R_1\{\ln[(R_1 + R_2)/2R_1] + (k_1/k_2)\ln[2R_2/(R_1 + R_2)]\}} \tag{7}$$

and the normalized Nusselt number is given by

$$\frac{Nu}{Nu_{\mathrm{cond}}} = -\frac{R_1\{\ln[(R_1 + R_2)/2R_1] + (k_1/k_2)\ln[2R_2/(R_1 + R_2)]\}}{R_1 \ln(R_2/R_1)} \times \int_{-1/2}^{1/2} \frac{\partial \Theta_1}{\partial x}\bigg|_{x=-1/2} \mathrm{d}y \tag{8}$$

When the normalized Nusselt numbers are plotted as a function of the Rayleigh number Ra_1 (refer to Ref. 9), it is clearly observed that the normalized Nusselt number for a layered annulus of $K_1/K_2 < 1$ is always greater than that of a homogeneous one, whereas it is constantly less for an annulus of $K_1/K_2 > 1$. When taking into account the effect of nonuniform thermal conductivity, it is found that the normalized Nusselt number for an annulus of $k_1/k_2 > 1$ is higher than that of a uniform one for $K_1/K_2 \leq 1$, whereas it is smaller for $K_1/K_2 > 1$. From the results thus obtained, it can be concluded that heat transfer from a layered annulus can be effectively minimized by selecting an outer layer material that has a lower permeability ($K_1/K_2 > 1$) and a smaller thermal conductivity ($k_1/k_2 > 1$). With this combination of the material properties, the convective flow in the inner region is greatly weakened. As a result, convective heat loss is significantly reduced and the ultimate goal of a pipe insulation is reached. For applications in the environmental protection, this is also the most desirable situation, since contamination, if occurred, will be essentially confined to the inner region.

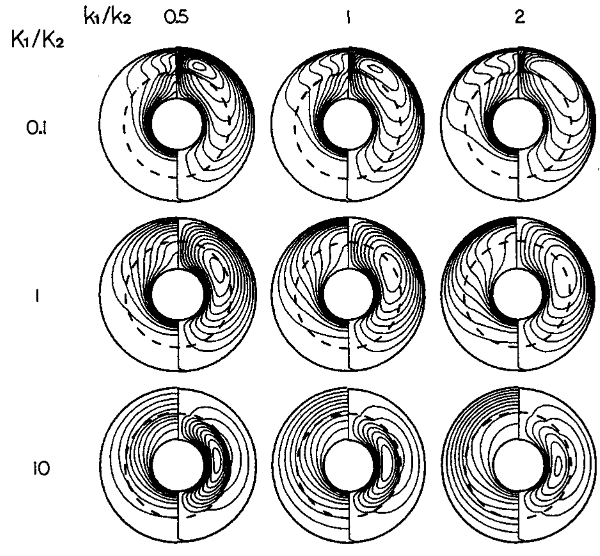

Fig. 2 Effects of conductivity ratio on the flow and temperature fields of a layered porous annulus ($Ra_1 = 100$, $\Delta\Theta = 0.1$, $\Delta\psi = 1$ for $K_1/K_2 = 0.1$ and 1, $\Delta\psi = 2$ for $K_1/K_2 = 10$).

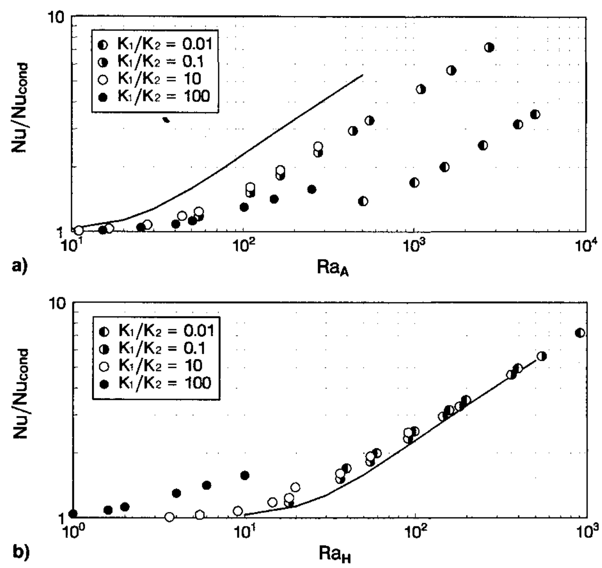

Fig. 3 Heat transfer results based on the effective permeability model: a) arithmetic and b) harmonic mean.

For engineering applications, it is a common practice to assume an effective permeability in all the design calculations. The effective permeability is usually based on an arithmetic mean

$$K_A = (K_1 + K_2)/2 \tag{9}$$

or a harmonic mean

$$K_H = 2K_1K_2/(K_1 + K_2) \tag{10}$$

While this approach has greatly simplified the problem, the justification of such a practice has never been verified. In the previous study,[7] the use of a harmonic mean has been suggested. Clearly, one major drawback of this approach is that it completely ignores the differences in the layer structure. For example, one would obtain the same effective permeability for two layered annuli in which the inner and outer sublayers are interchanged. As a consequence, the heat transfer results of these two annuli will be exactly the same if the approach described above is employed. This is exactly what has been shown in the previous study[7]: the heat transfer results are insensitive to the layer structure of the system, which, of course, has contradicted the results obtained here.

Despite its insufficiency to resolve the layer structures, the effective permeability model has been used widely in engineering design for its simplicity. Thus, there is a compelling need to evaluate the applicable range of the model. To this end, heat transfer results for the case of $k_1/k_2 = 1$ are plotted in Fig. 3 where the Rayleigh numbers are based on the effective permeability defined in Eqs. (9) and (10), respectively. While the symbols represent the results obtained from the layer model, the solid line is the result using an effective permeability. It is clear that the effective permeability model using harmonic means, although unable to reveal the characteristics of a layered structure, has a better agreement in the predicted heat transfer results. The model based on the arithmetic mean, on the other hand, considerably overestimates the heat transfer results. However, it should be noticed that, as the permeability contrast sharpens (particularly for $K_1/K_2 >> 1$), even the harmonic mean model cannot predict the heat transfer results accurately.

Conclusions

Natural convection in horizontal-layered porous annuli has been numerically investigated. The applicability of the effective permeability model in the calculation of heat transfer results of a nonuniform system has also been examined. From the present results, it is found that the normalized Nusselt number of a layered annulus with $K_1/K_2 < 1$ is always greater than that of a homogeneous one, whereas it is constantly less for an annulus with $K_1/K_2 > 1$. It is also found that the heat transfer prediction using a harmonic average permeability usually gives a better result than that of using an arithmetic average. However, as the permeability contrast increases ($K_1/K_2 >> 1$), the harmonic mean permeability model also fails. The results thus obtained are useful in the design of insulation systems and nuclear waste repositories. Although each application may have a different emphasis (e.g., minimal heat loss for pipe insulation and least contamination for waste disposal), the present results suggest that, by a careful matching of the sublayer materials, most requirements can be met easily.

Acknowledgment

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Low Prandtl Number Marangoni Convection with a Deforming Interface

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Introduction

THERMOCAPILLARY flows are driven by tangential shear associated with temperature-induced surface tension gradients at the interface between two immiscible fluids. In general, this phenomenon continues to generate interest because unsteady convection in crystal growth melts has a negative impact on solid morphology. Unsteady thermocapillary convection calculations for low *Pr* fluids were performed by Ohnishi and Azuma[1] in a rectangular cavity with an imposed flat-free surface. Their results indicated a transition from steady to unsteady flow at high $Ma \approx 400$, which corresponded to $Re \approx 2.67 \times 10^4$. These results were highly grid dependent in that the critical *Ma* decreased with each finer grid utilized and were in contrast to steady results calculated by Hadid and Roux[2] where the flow remained steady up to $Ma = 750$. Chen and Hwu[3] performed a numerical solution of both the flowfield and interface for $Pr = 0.01$ and fixed AR $A = 2$. These results indicated a bifurcation at very small *Ma* and with significant free surface deformation. It was suggested that *Ca* and critical *Ma* were inversely related by an exponential relationship and that a flat interface would undergo no transition to unsteady flow. However, Liakopoulos and Brown[4] calculated only steady thermocapillary convection in square cavities with *Ca* as large as 0.05.

The present work considers thermocapillary convection in a cavity and incorporates a deforming interface using two different approaches. The first is an asymptotic expansion with respect to the small parameter *Ca* where we develop $\mathcal{O}(1)$ and $\mathcal{O}(Ca)$ solutions. The second approach employs a linearized free surface condition. Details of the formulations are presented with both approaches predicting identical steady states for all parameter values considered.

Mathematical Model

The physical model consists of a rectangular calculation domain with A = width/height. There is a free surface ($y = H$) across which no mass transport takes place. The static contact angle is taken as 90 deg, which implies a flat equilibrium interface. A driving temperature difference ($\Delta T = T_h - T_c$) is imposed in the x direction by assuming differentially heated vertical walls. Adiabatic conditions are assumed at the

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